

# NLO EW corrections to $\text{pp} \rightarrow WW \rightarrow 4f$ in double-pole approximation at the LHC

Marina Billoni



in collaboration with S. Dittmaier, B. Jäger and C. Speckner

based on JHEP 1312 (2013) 043 [[arXiv:1310.1564 \[hep-ph\]](https://arxiv.org/abs/1310.1564)]

LoopFest XIII  
New York City College of Technology  
18 June 2014

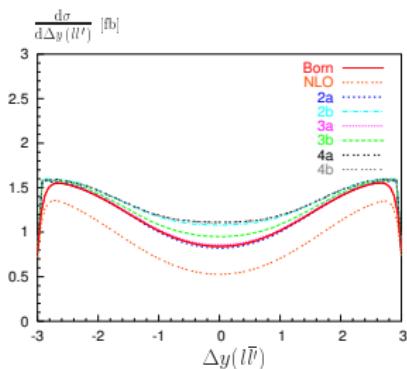
# Motivation

## $W^+W^-$ production at the LHC

- probe non-abelian structure of the Standard Model
- measurement of anomalous gauge couplings
- irreducible background to  $H \rightarrow WW^*$

For precise theoretical predictions we need

- QCD corrections
- EW corrections (enhancement by **Sudakov logarithms** at high scales)



EW corrections could fake  
New Physics!

[Accomando, Kaiser '05]  
(logarithmic approximation)

# Recent results

## QCD corrections

- NLO QCD corrections (+ parton shower matching @NLO)  
[Cascioli, Hoeche, Krauss, Maierhofer, Pozzorini, Siegert '13]
- approximate NNLO QCD results  
[Dawson, Lewis, Zeng '13, Campanario, Rauch, Sapeta '13]
- step towards NNLO QCD corrections  
numerical results only valid in high energy limit  
[Chachamis, Czakon, Eiras '08]

## Fixed order calculations including EW corrections

- one-loop corrections to on-shell  $W^+W^-$  production  
[Bierweiler, Kasprzik, Kühn, Uccirati '12/13]
- one-loop corrections to on-shell  $W^+W^-$  production including  
 $\gamma q$ -induced contributions [Baglio, Ninh, Weber '13]
  - ▶ EW corrections for integrated cross sections:  $\mathcal{O}(1\%)$
  - ▶ EW corrections for distribution at high scales: up to  $-40\%$

# Our calculation: RacoonWW approach

RacoonWW:  $e^+ e^- \rightarrow 4 \text{ fermions}$  in double-pole approximation

[Denner, Dittmaier, Roth, Wackerlohe '00]

Leading order: full off-shell calculation

- $\bar{q}q$ -induced contribution ( $q = u, d, c, s$ )

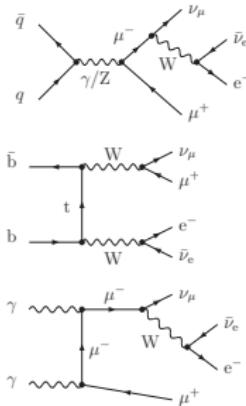
$$\bar{q}q \rightarrow \nu_\mu \mu^+ e^- \bar{\nu}_e$$

- $\bar{b}b$ -induced contribution (< 2% @LO)

$$\bar{b}b \rightarrow \nu_\mu \mu^+ e^- \bar{\nu}_e$$

- $\gamma\gamma$ -induced contribution (< 1% @LO)

$$\gamma\gamma \rightarrow \nu_\mu \mu^+ e^- \bar{\nu}_e$$

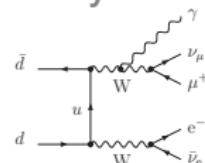


EW corrections to  $\bar{q}q$ -induced contribution only

- **Virtual** corrections:

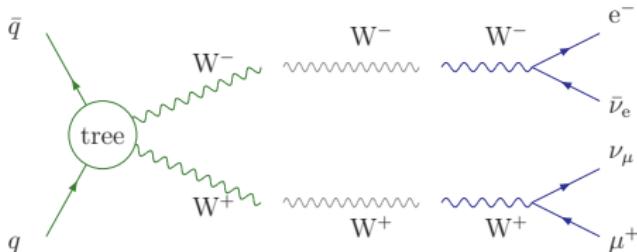
Double-pole approximation (DPA)

- **Real** corrections: full off-shell calculation



# Double Pole Approximation: Basic Idea

LO



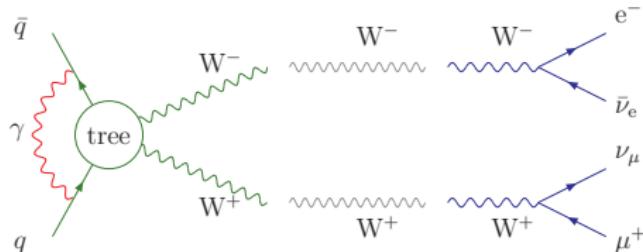
on-shell production   Breit-Wigner   on-shell decay

$$\begin{aligned} \mathcal{M}_{\text{Born}, \text{DPA}} &= \sum_{\lambda_+, \lambda_-} \frac{1}{[k_+^2 - M_W^2 + iM_W\Gamma_W] [k_-^2 - M_W^2 + iM_W\Gamma_W]} \\ &\times \left\{ \mathcal{M}_{\text{Born}}^{\bar{q}q \rightarrow W^+W^-} \mathcal{M}_{\text{Born}}^{W^+ \rightarrow \nu_\mu \mu^+} \mathcal{M}_{\text{Born}}^{W^- \rightarrow e^- \bar{\nu}_e} \right\} \end{aligned}$$

- consider on-shell production of W pair and on-shell W decay
- add off-shell propagators including Breit-Wigner distribution
- sum over the polarization states
- for gauge invariance: on-shell projection of momenta needed!
- error estimate:  $\Gamma_W/M_W$

# Double Pole Approximation: Basic Idea

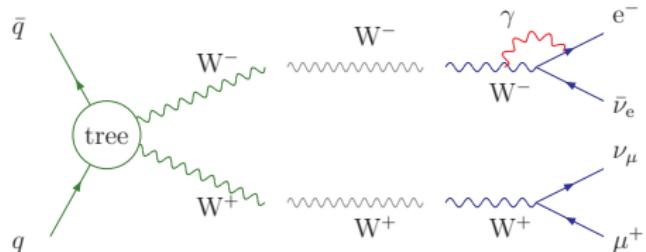
NLO



One distinguishes between **factorizable** (corrections to the decay OR to the production) and non-factorizable corrections (only soft photon exchange).

# Double Pole Approximation: Basic Idea

NLO

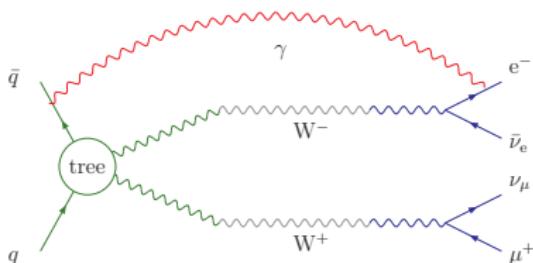


on-shell production    Breit-Wigner    on-shell decay

One distinguishes between **factorizable** (corrections to the decay OR to the production) and non-factorizable corrections (only soft photon exchange).

# Double Pole Approximation: Basic Idea

NLO



on-shell production    Breit-Wigner    on-shell decay

One distinguishes between factorizable (corrections to the decay OR to the production) and **non-factorizable** corrections (only soft photon exchange).

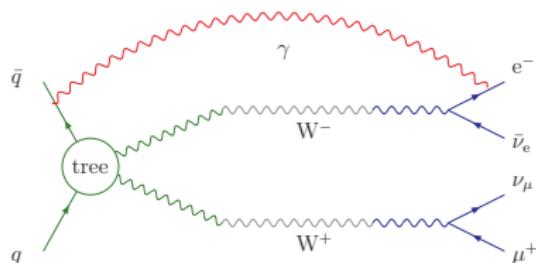
- selection of all doubly resonant one-loop diagrams to  $p\bar{p} \rightarrow \nu_\mu \mu^+ e^- \bar{\nu}_e$
- expansion around the poles
- error estimate:  $\alpha/\pi \times \Gamma_W/M_W$

## Advantages

- # diagrams reduced
- diagrams much easier to calculate (without DPA EW corrections to  $2 \rightarrow 4$  process!) → faster code

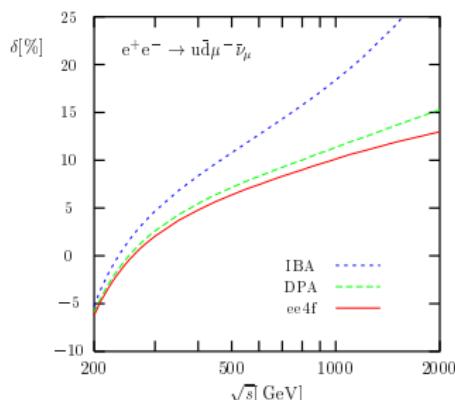
# Double Pole Approximation: Basic Idea

NLO



on-shell production    Breit-Wigner    on-shell decay

One distinguishes between factorizable (corrections to the decay OR to the production) and non-factorizable corrections (only soft photon exchange).



[Denner, Dittmaier, Roth, Wieders '06]

full off-shell calculation of  $e^+e^- \rightarrow 4f$

# Virtual corrections in DPA

$$\int_4 d\hat{\sigma}_{\bar{q}q}^{\text{virt,DPA}} \sim \int_4 d\Phi_4 \left\{ 2 \operatorname{Re} \left[ \left( \mathcal{M}_{\text{Born,DPA}}^{\bar{q}q \rightarrow WW \rightarrow \nu_\mu \mu^+ e^- \bar{\nu}_e} \right)^* \delta \mathcal{M}_{\text{virt,fact,DPA}}^{\bar{q}q \rightarrow WW \rightarrow \nu_\mu \mu^+ e^- \bar{\nu}_e} \right] + \left| \mathcal{M}_{\text{Born,DPA}}^{\bar{q}q \rightarrow WW \rightarrow \nu_\mu \mu^+ e^- \bar{\nu}_e} \right|^2 \delta_{\text{nfact}}^{\text{virt}} \right\}$$

## Factorizable Corrections

$$\delta \mathcal{M}_{\text{virt,fact,DPA}}^{\bar{q}q \rightarrow W^+ W^- \rightarrow \nu_\mu \mu^+ e^- \bar{\nu}_e} = \sum_{\lambda_+, \lambda_-} \frac{1}{[k_+^2 - M_W^2 + iM_W\Gamma_W] [k_-^2 - M_W^2 + iM_W\Gamma_W]} \\ \times \left\{ \delta \mathcal{M}_{\text{Born}}^{\bar{q}q \rightarrow W^+ W^-} \mathcal{M}_{\text{Born}}^{W^+ \rightarrow \nu_\mu \mu^+} \mathcal{M}_{\text{Born}}^{W^- \rightarrow e^- \bar{\nu}_e} \right. \\ \left. + \mathcal{M}_{\text{Born}}^{\bar{q}q \rightarrow W^+ W^-} \delta \mathcal{M}_{\text{Born}}^{W^+ \rightarrow \nu_\mu \mu^+} \mathcal{M}_{\text{Born}}^{W^- \rightarrow e^- \bar{\nu}_e} \right. \\ \left. + \mathcal{M}_{\text{Born}}^{\bar{q}q \rightarrow W^+ W^-} \mathcal{M}_{\text{Born}}^{W^+ \rightarrow \nu_\mu \mu^+} \delta \mathcal{M}_{\text{Born}}^{W^- \rightarrow e^- \bar{\nu}_e} \right\}$$

## Non-factorizable corrections

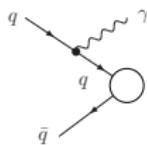
- finite part: very small (for  $e^+ e^- \rightarrow 4f$ ) [Denner, Dittmaier, Roth '97]
- singular part: important for cancellation of IR singularities

# Real corrections: $\bar{q}q \rightarrow \nu_\mu \mu^+ e^- \bar{\nu}_e \gamma$

## Subprocesses with final-state photon

- Dipole subtraction formalism [Dittmaier '99]

$$\int_5 d\hat{\sigma}_{\bar{q}q}^{\text{real}} + \int_0^1 dx \int_4 d\hat{\sigma}_{\bar{q}q}^{\text{fact}} = \underbrace{\int_5 \left[ d\hat{\sigma}_{\bar{q}q}^{\text{real}} - d\hat{\sigma}_{\bar{q}q}^{\text{real,sing}} \right]}_{\text{IR finite}} + \int_4 \left[ \int_1 d\hat{\sigma}_{\bar{q}q}^{\text{real,sing}} \right] + \int_0^1 dx \int_4 d\hat{\sigma}_{\bar{q}q}^{\text{fact}}$$



- ▶ Convolution part with collinear IS singularities absorbed into PDFs (Factorization)
- ▶ Endpoint contribution for cancellation of IR singularities in virtual corrections

# Cancellation of IR singularities

Soft and collinear singularities are regularized by introducing an infinitesimal photon mass  $\lambda$  and small fermion masses  $m_i$ .

(checked against pure dim. regularization)

→ cancellation between virtual and real corrections

## Endpoint contribution

$$\int_5 d\hat{\sigma}_{ab}^{\text{real,sing}} = \int_0^1 dx \int_4 d\hat{\sigma}_{ab}^{\text{real,conv}} + \int_4 d\hat{\sigma}_{ab}^{\text{real,endp}}$$

$$\int_4 d\hat{\sigma}_{\bar{q}q}^{\text{virt,fin}} = \int_4 d\hat{\sigma}_{\bar{q}q}^{\text{virt}} + \int_4 d\hat{\sigma}_{\bar{q}q}^{\text{real,endp}}$$

$$d\hat{\sigma}_{ab}^{\text{real,endp,DPA}} =$$

$$- d\hat{\sigma}_{ab}^{\text{Born,DPA}} \frac{\alpha}{2\pi} \sum_{i=1}^6 \sum_{j=i+1}^6 (-1)^{i+j} Q_i Q_j \left[ \mathcal{L}(\hat{\mathbf{s}}_{ij}, m_i^2) + \mathcal{L}(\hat{\mathbf{s}}_{ij}, m_j^2) + \text{const.} \right]$$

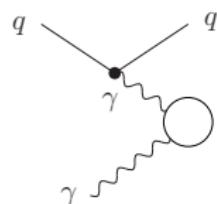
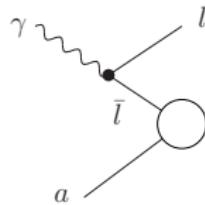
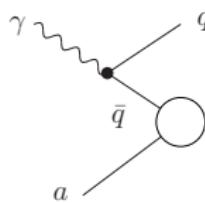
$$\mathcal{L}(s, m^2) = \ln\left(\frac{m^2}{s}\right) \ln\left(\frac{\lambda^2}{s}\right) + \ln\left(\frac{\lambda^2}{s}\right) - \frac{1}{2} \ln^2\left(\frac{m^2}{s}\right) + \frac{1}{2} \ln\left(\frac{m^2}{s}\right)$$

→ IR-finite!

# Real corrections: $\gamma q \rightarrow \nu_\mu \mu^+ e^- \bar{\nu}_e q$

Photon-quark induced subprocesses  
no soft, only collinear singularities occur

- $\gamma \rightarrow \bar{q}q$ : dipole subtraction
- $\gamma \rightarrow \bar{l}l$ : cut on transverse momentum of charged leptons
- $q \rightarrow \gamma^* q$ : effective collinear factor
  - restores regulator mass dependence in singular limit
  - mass dependence drops out by redefinition of pdfs



# Further details on our calculation

- for  $\sqrt{\hat{s}} < 2M_W + x$ : Improved Born Approximation  
[Denner, Dittmaier, Roth, Wackerlo '01]
- Matrix elements based on RacoonWW
- all LO MEs checked against Madgraph
- multi-channel integrator based on Coffey $\gamma\gamma$  for  $\gamma\gamma \rightarrow 4f$   
[Bredenstein, Dittmaier, Roth '05]
- Tools for virtuals: In general we use
  - ▶ FeynArts
  - ▶ inhouse Mathematica routines
  - ▶ Collier [Denner, Dittmaier, Hofer]  
→ Fortran code
- Setup for independent check only:
  - ▶ FeynArts, FormCalc, LoopTools [Hahn et al.]

## Our calculation: Summary

$$\begin{aligned}\sigma_{\text{pp}}^{\text{NLO}} &= \int_0^1 dx_1 \int_0^1 dx_2 \left\{ \left( \sum_{q=u,d,c,s} f_q(x_1, \mu_F) f_{\bar{q}}(x_2, \mu_F) \right. \right. \\ &\quad \times \left[ \int_4 d\hat{\sigma}_{\bar{q}q}^{\text{LO}} + \int_4 d\hat{\sigma}_{\bar{q}q}^{\text{virt}} + \int_5 d\hat{\sigma}_{\bar{q}q}^{\text{real}} + \int_0^1 dx \int_4 d\hat{\sigma}_{\bar{q}q}^{\text{fact}} \right] \\ &\quad \left. \left. + (q \leftrightarrow \bar{q}) \right) \rightarrow \delta_{\bar{q}q} \right.\end{aligned}$$

# Our calculation: Summary

$$\begin{aligned}\sigma_{\text{pp}}^{\text{NLO}} &= \int_0^1 dx_1 \int_0^1 dx_2 \left\{ \left( \sum_{q=u,d,c,s} f_{\bar{q}}(x_1, \mu_F) f_q(x_2, \mu_F) \right. \right. \\ &\quad \times \left[ \int_4 d\hat{\sigma}_{\bar{q}q}^{\text{LO}} + \int_4 d\hat{\sigma}_{\bar{q}q}^{\text{virt}} + \int_5 d\hat{\sigma}_{\bar{q}q}^{\text{real}} + \int_0^1 dx \int_4 d\hat{\sigma}_{\bar{q}q}^{\text{fact}} \right] + (q \leftrightarrow \bar{q}) \Bigg) \\ &\quad \rightarrow \delta_{\bar{q}q} \\ &+ f_\gamma(x_1, \mu_F) \sum_{q=u,d,c,s} \left( f_q(x_2, \mu_F) \left[ \int_5 d\hat{\sigma}_{\gamma q}^{\text{real}} + \int_0^1 dx \int_4 d\hat{\sigma}_{\gamma q}^{\text{fact}} \right] + (q \leftrightarrow \bar{q}) \right) \\ &+ f_\gamma(x_2, \mu_F) \sum_{q=u,d,c,s} \left( f_{\bar{q}}(x_1, \mu_F) \left[ \int_5 d\hat{\sigma}_{\bar{q}\gamma}^{\text{real}} + \int_0^1 dx \int_4 d\hat{\sigma}_{\bar{q}\gamma}^{\text{fact}} \right] + (\bar{q} \leftrightarrow q) \right) \\ &\quad \rightarrow \delta_{\gamma q}\end{aligned}$$

# Our calculation: Summary

$$\begin{aligned}
\sigma_{\text{pp}}^{\text{NLO}} &= \int_0^1 dx_1 \int_0^1 dx_2 \left\{ \left( \sum_{q=u,d,c,s} f_{\bar{q}}(x_1, \mu_F) f_q(x_2, \mu_F) \right. \right. \\
&\quad \times \left[ \int_4 d\hat{\sigma}_{\bar{q}q}^{\text{LO}} + \int_4 d\hat{\sigma}_{\bar{q}q}^{\text{virt}} + \int_5 d\hat{\sigma}_{\bar{q}q}^{\text{real}} + \int_0^1 dx \int_4 d\hat{\sigma}_{\bar{q}q}^{\text{fact}} \right] + (q \leftrightarrow \bar{q}) \Bigg) \\
&\quad \rightarrow \delta_{\bar{q}q} \\
&\quad + f_\gamma(x_1, \mu_F) \sum_{q=u,d,c,s} \left( f_q(x_2, \mu_F) \left[ \int_5 d\hat{\sigma}_{\gamma q}^{\text{real}} + \int_0^1 dx \int_4 d\hat{\sigma}_{\gamma q}^{\text{fact}} \right] + (q \leftrightarrow \bar{q}) \right) \\
&\quad + f_\gamma(x_2, \mu_F) \sum_{q=u,d,c,s} \left( f_{\bar{q}}(x_1, \mu_F) \left[ \int_5 d\hat{\sigma}_{\bar{q}\gamma}^{\text{real}} + \int_0^1 dx \int_4 d\hat{\sigma}_{\bar{q}\gamma}^{\text{fact}} \right] + (\bar{q} \leftrightarrow q) \right) \\
&\quad \rightarrow \delta_{\gamma q} \\
&\quad + \left( f_b(x_1, \mu_F) f_b(x_2, \mu_F) \int_4 d\hat{\sigma}_{bb}^{\text{LO}} + (b \leftrightarrow \bar{b}) \right) \\
&\quad \rightarrow \delta_{\bar{b}b} \\
&\quad \left. + f_\gamma(x_1, \mu_F) f_\gamma(x_2, \mu_F) \int_4 d\hat{\sigma}_{\gamma\gamma}^{\text{LO}} \right\} \\
&\quad \rightarrow \delta_{\gamma\gamma}
\end{aligned}$$

# Numerical Setup

## Default Setup

- NNPDF2.3qed
- factorization scale  $\mu_F = M_W$
- photon recombination

## Minimal Cuts

- $p_{T,\ell} > 20 \text{ GeV}, |y_\ell| < 2.5$
- jet veto:  $p_{T,\text{jet}} > 100 \text{ GeV}$

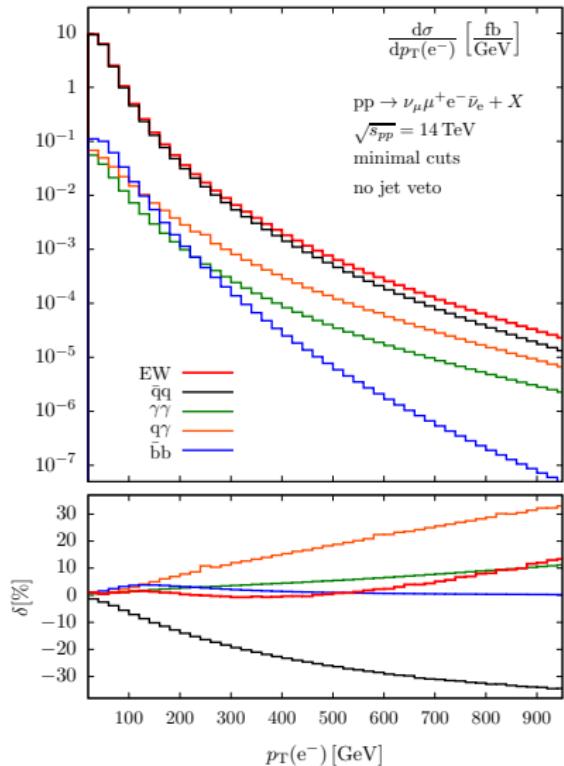
## ATLAS Cuts

- $p_{T,\ell} > 20 \text{ GeV}, |y_\ell| < 2.5$
- $p_{T,\ell}^{\text{leading}} > 25 \text{ GeV}, E_T^{\text{miss}} > 25 \text{ GeV}, R_{e\mu} > 0.1, M_{e\mu} > 10 \text{ GeV}$
- jet veto:  $p_{T,\text{jet}} > 25 \text{ GeV}$

	$\sigma_{\bar{q}q}^{\text{LO}} [\text{fb}]$	$\delta_{\bar{q}q} [\%]$	$\delta_{\gamma q} [\%]$	$\delta_{\gamma\gamma} [\%]$	$\delta_{\bar{b}b} [\%]$
LHC14	412.5	-2.7	0.6	0.7	1.7
LHC8	236.8	-2.8	0.5	0.8	0.9
ATLAS cuts	163.8	-3.0	-0.3	1.0	1.0

$\delta_x$ : relative correction by contribution x compared to  $\sigma_{\bar{q}q}^{\text{LO}}$

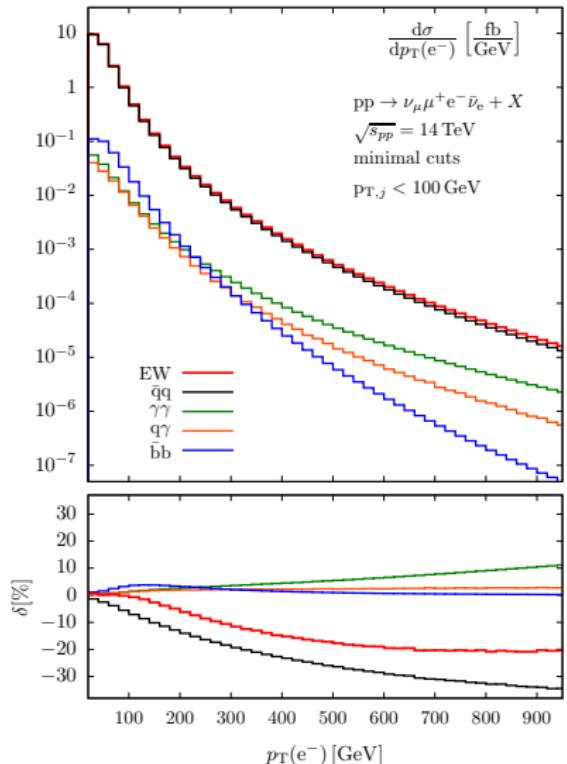
# LHC14: transverse momentum distribution



without jet veto

- $\delta_{\bar{q}q}$ :  $-30\%$  at  $p_T = 900 \text{ GeV}$   
(Sudakov logs)
- $\delta_{\gamma\gamma}$ : up to  $+10\%$
- $\delta_{\gamma q}$ : large due to **soft W emission**  
→ same effect in QCD corrections  
leads to huge  $K$ -factors  
→ apply jet veto

# LHC14: transverse momentum distribution



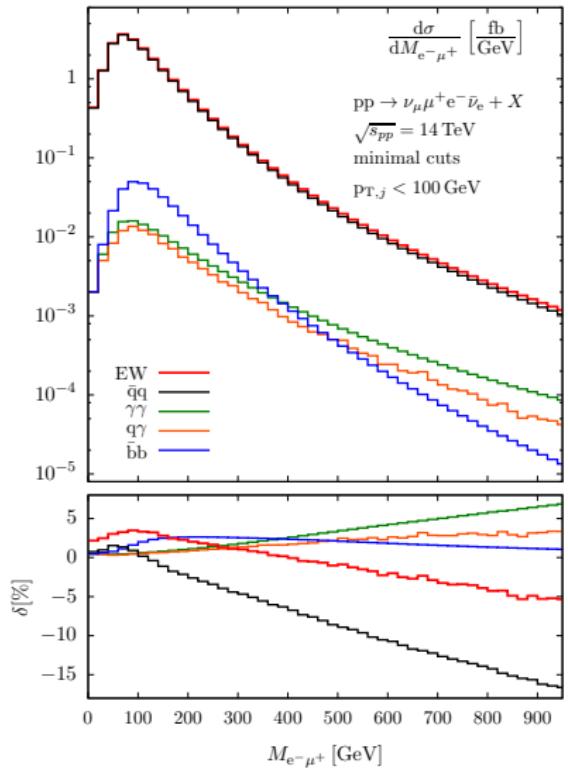
without jet veto

- $\delta_{\bar{q}q}$ :  $-30\%$  at  $p_T = 900 \text{ GeV}$   
(Sudakov logs)
- $\delta_{\gamma\gamma}$ : up to  $+10\%$
- $\delta_{q\gamma}$ : large due to **soft W emission**  
→ same effect in QCD corrections  
leads to huge  $K$ -factors  
→ apply jet veto

with jet veto  $p_{T,\text{jet}} > 100 \text{ GeV}$

- $\delta_{q\gamma} < 5\%$  even in the region of high transverse momentum
- $\delta_{EW} (= \delta_{\bar{q}q} + \delta_{q\gamma} + \delta_{\gamma\gamma} + \delta_{\bar{b}b})$   
=  $-20\%$  at  $p_T = 900 \text{ GeV}$

# LHC14: invariant mass distribution of charged leptons



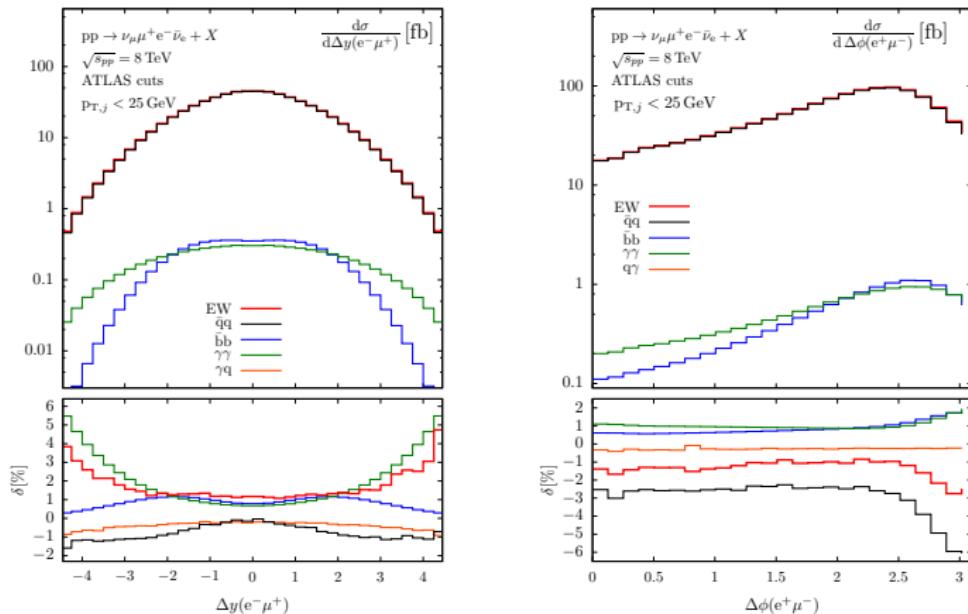
with jet veto

- relatively large negative EW corrections partially compensated by positive contributions, especially  $\delta_{\gamma\gamma}$

→ moderate corrections even at high invariant mass (< 10%)

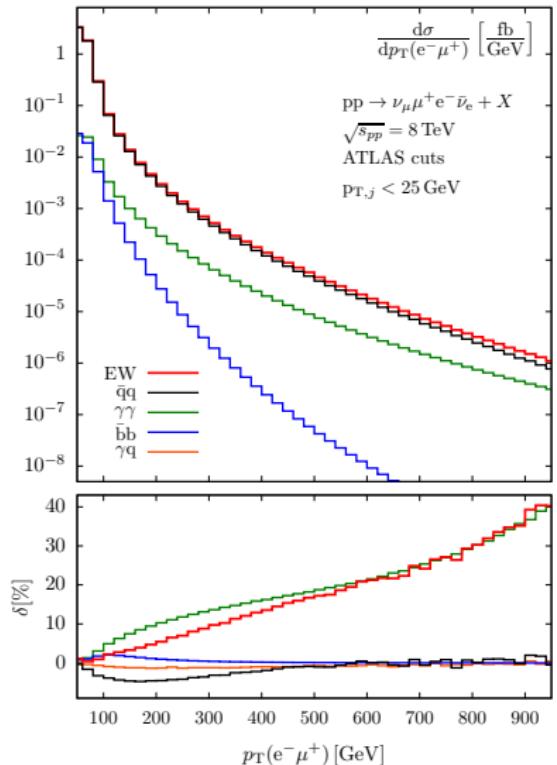
- same behaviour found for transverse-mass distribution  $M_{T,WW}$

# ATLAS cuts: angular distributions



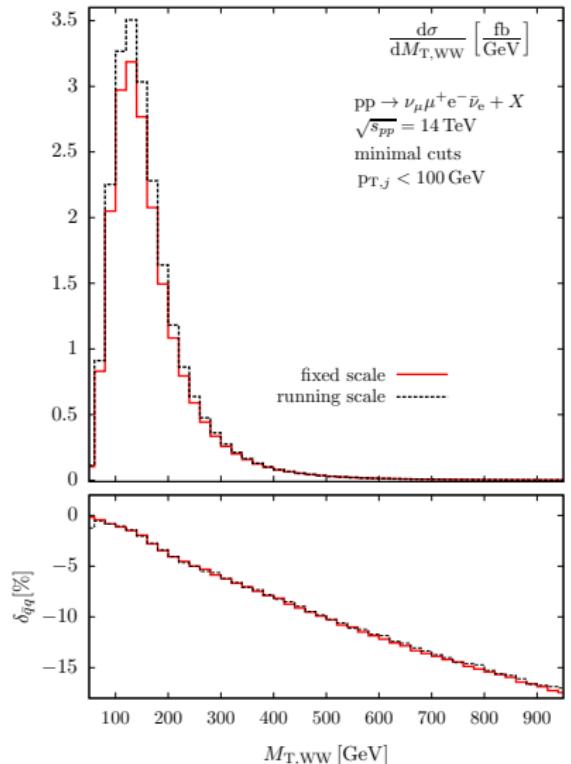
- angular distributions in general less affected by EW corrections
- $\gamma\gamma$  contribution dominates the forward/backward emission of the charged leptons (reported by Bierweiler et al.)

# ATLAS cuts: transverse momentum $p_T(e^-\mu^+)$



- real radiation of **hard** photon:  
large impact by  $\text{pp} \rightarrow W^+W^-\gamma$  with **hard** photon
- large photon recoil

# LHC14: Scale dependence



- variation of  $\mu_F$  by a factor 2 changes the inclusive cross section by around  $\pm 8\%$
- but  $\delta_{\bar{q}q}$  shows no scale dependence

fixed scale  $\mu_F = M_W$ ,  
running scale  $\mu_F = M_{WW}$

→ **factorization** of EW corrections:

$$d\sigma = d\sigma_{qq}^{\text{QCD}} \times (1 + \delta_{\bar{q}q}) + d\sigma_{gg} + d\sigma_{\gamma\gamma} + d\sigma_{\gamma q}$$

$d\sigma_{qq}^{\text{QCD}}$ : state-of the-art QCD prediction

# Conclusion + Outlook

## Conclusion

We have calculated the EW corrections to  $\nu_\mu \mu^+ e^- \bar{\nu}_e$  production at the LHC in double-pole approximation:

- first evaluation of EW corrections to W-pair production with decays (realistic event selection)
- previous (idealized) on-shell result confirmed within some %
  - ▶ EW corrections to integrated cross sections small
  - ▶ but sizable effects due to Sudakov logarithms in distributions at high scale
  - ▶  $\gamma\gamma$  contribution has to be taken into account
  - ▶  $\gamma q$  contribution is suppressed by jet veto
  - ▶ scale dependence low
- first step towards calculation of full EW corrections to 4f production

## Outlook

- full EW and QCD corrections to  $\nu_\mu \mu^+ e^- \bar{\nu}_e$  production at the LHC
- implementation of subtraction terms for non-collinear-safe observables  
[Dittmaier, Kabelschacht, Kasprzik '08]

Backup slides

# Improved Born Approximation

$$d\hat{\sigma}_{\bar{q}q}^{\text{IBA}} = F(x_1 x_2) d\Phi_4 \overline{\sum} \left| \mathcal{M}_{\text{IBA}}^{\bar{q}q \rightarrow \nu_\mu \mu^+ e^- \bar{\nu}_e} \right|^2 [1 + \delta_{\text{Coul}}(\hat{s}, k_+^2, k_-^2)] g(\bar{\beta})$$

Coulomb singularity [Fadin et al., Beenakker et al.]:

$$\begin{aligned}\delta_{\text{Coul}}(\hat{s}, k_+^2, k_-^2) &= \frac{\alpha(0)}{\bar{\beta}} \text{Im} \left\{ \ln \left( \frac{\beta - \bar{\beta} + \Delta_M}{\beta + \bar{\beta} + \Delta_M} \right) \right\}, \\ \bar{\beta} &= \frac{\sqrt{\hat{s}^2 + k_+^4 + k_-^4 - 2\hat{s}k_+^2 - 2\hat{s}k_-^2 - 2k_+^2k_-^2}}{\hat{s}}, \\ \beta &= \sqrt{1 - \frac{4(M_W^2 - iM_W\Gamma_W)}{\hat{s}}}, \quad \Delta_M = \frac{|k_+^2 - k_-^2|}{\hat{s}}\end{aligned}$$

damping factor:

$$g(\bar{\beta}) = (1 - \bar{\beta}^2)^2$$

# On-shell projection

[Denner et al.]

$$a(p_+) + b(p_-) \rightarrow f_1(k_1) + \bar{f}_2(k_2) + f_3(k_3) + \bar{f}_4(k_4)$$

- fix direction of  $W^+$  boson, of fermion  $f_1$ , and of fermion  $f_3$

$$\hat{k}_{+0} = \frac{1}{2}\sqrt{s}, \quad \mathbf{k}_+ = \frac{\mathbf{k}_+}{|\mathbf{k}_+|}\beta\frac{\sqrt{s}}{2}, \quad \hat{k}_-^\mu = p_+^\mu + p_-^\mu - \hat{k}_+^\mu,$$

$$\hat{k}_1^\mu = k_1^\mu \frac{M_W^2}{2\hat{k}_+ k_1}, \quad \hat{k}_2^\mu = \hat{k}_+^\mu - \hat{k}_1^\mu,$$

$$\hat{k}_3^\mu = k_3^\mu \frac{M_W^2}{2\hat{k}_- k_3}, \quad \hat{k}_4^\mu = \hat{k}_-^\mu - \hat{k}_3^\mu$$

$$\text{with } \beta = \sqrt{1 - 4M_W^2/s}$$

$$\rightarrow \hat{k}_+^2 = \hat{k}_-^2 = M_W^2$$

## Effective collinear factor

$$f(m_q, x, E_q, \theta) =$$

$$\frac{\sin^2 \frac{\theta}{2}}{\left[ \sin^2 \frac{\theta}{2} + \frac{m_q^2 x^2}{4E_q^2(1-x)^2} \right]^2} \left\{ \sin^2 \frac{\theta}{2} + \frac{m_q^2 x^4}{4E_q^2(1-x)^2(1+(1-x)^2)} \right\}$$

where  $\theta$  is the angle between the incoming and the outgoing quark of mass  $m_q$  defined in the partonic centre-of-mass frame,  $E_q$  is the energy of the incoming quark in the same frame, and  $x = k^0/E_q$  is the fraction of the incoming quark's momentum that is carried by the emitted photon.